Example. *A company manufactures steel bolts that continuously feed an assembly line downstream. Historically, the thickness of these bolts follows a normal distribution with a standard deviation of 1.6 mm. A recent random sample of 10 observations yielded these values of thickness, in mm*
9.7,9.9,10.3,10.1,10.5,9.4,9.9,10.1,9.7,10.3

(a) Construct a 95% confidence interval for the true mean thickness,

(b) Find the number of observations, n, required to estimate the true mean thickness to within ±0.2 mm using 95% confidence interval.

Solution. (a) Given that $n = 10$, $\overline{X} = 9.99$, $\sigma = 1.6$. The confidence level is $1 - \alpha = 0.95$.

Step 1 Specify the confidence level $(1 - \alpha)$
Confidence level $= 1 - \alpha = 0.95$, thus $\alpha = 0.05$.

Step 2 Given information
Specify the given information, sample size $n = 10$, sample mean $\overline{X} = 9.99$.

Step 3 Specify the formula
$100(1 - \alpha)\%$ confidence interval for the population mean μ is

$$\overline{X} - E \leq \mu \leq \overline{X} + E$$

where $E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

Step 4 Determine the critical value
The critical value $Z_{\alpha/2} = Z_{0.025} = 1.96$.

Step 5 Compute the margin of error
The margin of error for mean is

$$\begin{aligned} E &= Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \\ &= 1.96\frac{1.6}{\sqrt{10}} \\ &= 0.992. \end{aligned}$$

Step 6 Determine the confidence interval 95% confidence interval estimate for population mean is

$$\begin{aligned} &\overline{X} - E \leq \mu \leq \overline{X} + E \\ &9.99 - 0.992 \leq \mu \leq 9.99 + 0.992 \\ &8.998 \leq \mu \leq 10.982. \end{aligned}$$

Thus, 95% confidence interval estimate for population mean is $(8.998, 10.982)$.

(b) Given that the standard deviation is $\sigma = 1.6$, margin of error $E = \pm 0.2$. The confidence coefficient is $1 - \alpha = 0.95$. Thus $\alpha = 0.05$.

The critical value of Z is $z = Z_{\alpha/2} = 1.96$.
The minimum sample size required to estimate the mean is

$$\begin{aligned}
n &= \left(\frac{z * \sigma}{E}\right)^2 \\
&= \left(\frac{1.96 * 1.6}{0.2}\right)^2 \\
&= 245.8624 \\
&\approx 246
\end{aligned}$$

Thus, the sample of size $n = 246$ will ensure that the 0.95% confidence interval for the mean will have a margin of error ± 0.2.